Monthly Contest 4
Due March 13, 2020

Instructions

This contest consists of 6 problems, each worth 5 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problem's solution (not the total pages of all the solutions). Please email all issues to ethan.tang003@gmail.com for clarification.

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any website (including forums) unless otherwise stated in the problem.**

Problems

1. Prove that for every positive integer n there exists an n-digit number divisible by 5^n all of whose digits are odd.

2. There is a 2020 by 2020 grid of spaces. When placing N colored dots in N spaces, a right triangle with vertices at colored spaces and sides parallel to the sides of the grid is guaranteed to exist. Find the minimum possible value of N.

3. In an $m \times n$ rectangular grid, where m and n are odd integers, 1×2 dominoes are initially placed so as to exactly cover all but one of the

unit squares at one corner of the grid. By sliding a domino to the empty space, you can expose another square. Show that you can move the empty square to any corner of the rectangle.

4. In acute triangle ABC, CF is an altitude, with F on AB, and BM is a median, with M on CA. Given that $BM = CF$ and $\angle MBC = \angle FCA$ prove that the triangle ABC is equilateral.

5. Given a finite number of squares with total area 4, prove that you can cover a unit square with them.

6. Find all real polynomials $P(x)$ such that $P(x\sqrt{2}) = P(x + \sqrt{1 - x^2})$ for all $|x| \leq 1$