Monthly Contest 2
Due December 20, 2019

Instructions

This contest consists of 6 problems, each worth 5 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problem's solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any website (including forums) unless otherwise stated in the problem.**

Problems

1. On a piece of paper, I have written 100 statements, "There is at least one incorrect statement on this paper", "There are exactly two incorrect statements on this paper", so on until "There are exactly one hundred incorrect statements on this paper", alternating between "at least" and "exactly". Which, if any, of these statements are true?

2. I have a special calculator. It has one operator, the @ operator, given by $A \ @\ B = 1 - \frac{A}{B}$. Prove that I can use the calculator to do division, multiplication, subtraction.

3. I have a standard 52-card deck. I draw cards from the deck one by one, without putting them back in the deck. Every time before drawing a card I guess the suit of the card. I always guess the suit that occurs most frequently in the remaining deck (if there are several such suits, I can choose any one of them). Prove that I will guess the right suit at least 13 times.

4. Set S has a finite N points in the plane, such that no three points are colinear. Given that for any three points A, B, C in S, there exists another point D in S such that A, B, C, D (in some order) are the vertices of a parallelogram, find and prove the maximum possible value of N.

5. Let
$$S = 1 + \frac{1}{1+\frac{1}{3}} + \frac{1}{1+\frac{1}{3}+\frac{1}{6}} ... + \frac{1}{\frac{1}{3}+\frac{1}{6}+...+\frac{1}{1993006}}$$
where each denominator is a partial sum of the reciprocals of triangular numbers. Prove that $S > 1001$.

6. In the country of Ginoksberg there are several cities and several roads. Every road connects exactly 2 cities. Each city is connected to at least 3 other cities. Prove that there is a cycle such that the number of cities in it is not divisible by 3.