

Monthly Contest 4
Due April 5, 2019

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any website (including forums) unless otherwise stated in the problem.**

Problems

1. Prove that if x, y are integers not divisible by 3, then $(x^2 - y^2)$ is divisible by 3.
2. How many ordered triples of positive numbers (a, b, c) are there such that $\sqrt{a^b + c!} = 28$?
3. How many subsets S of $1, 2, 3, \dots, 10$ have the property that there does not exist $a, b \in S$ such that ab is a square?

4. Line segment \overline{AC} has length 30. Point B is on AC such that $AB = 9$. Let D be a point such that ADC is a non-degenerate isosceles triangle, and both AD and BD have integer lengths. Find the sum of all possible perimeters of triangle ADC .
5. I roll a fair 6-sided dice until I get the sequence "2-4-5" in 3 consecutive rolls. Find the probability that I roll the dice an odd number of times.