**Monthly Contest 3**
**Due January 25, 2017**

## Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. You may view any book or website (including forums) as long as you do not communicate the problem in any sort of manner (e.g. posting the problem statement or variations on a forum).

## Problems

1. Xavier, Yvonne, and Zach play badminton. Two players play at a time, and the third person plays the winner of the previous game (ties are not possible). Eventually, Xavier played 10 games while Yvonne played 21 games. How many games did Zach play?

2. The lengths of the sides of an acute angled triangle ABC are consecutive integers with BC < CA < AB. Let BD be the altitude from B to CA. Prove that AD - CD = 4.

3. (ARML 2016 Team, ♯8) Compute the number of permutations $x_1, x_2, ..., x_{10}$ of the integers -3, -2, -1, . . . , 6 that satisfy the chain of inequalities $x_1x_2 \leq x_2x_3 \leq . . . \leq x_9x_{10}$.

4. Let $A_1A_2A_3...A_8$ be a regular octagon with O as its center. Triangular regions $OA_iA_{i+1}$, $1 \leq i \leq 8$ ($A_9 = A_1$) are to be colored red, blue, and green such that adjacent regions are colored in different colors. In how many ways can this be done?

5. Given a 20 by 20 board, prove whether or not it is possible to fill with 2 by 1 dominoes when 2 opposing corners of the board are removed.