EXPANDING FRACTIONS GENERATE HIDDEN INSIGHTS

The **repetend** of a rational number is the sequence of digits that repeat after the decimal point. For example, in \( \frac{1}{22} = .045454545 \ldots = .045 \), the **repetend** is 45.

1. Represent \( .037037 \ldots = .037 \) as a fraction. How about \( .6037037 \ldots = .6037 \) ?

2. Find the smallest positive integer \( n \) such that \( 1/n \) has a repetend of length 4.

3. An \( n \)-parasitic (or Dyson) number is a positive integer which can be multiplied by \( n \) by moving its rightmost digit to the beginning. Find \( n \)-parasitic numbers for \( n=4 \) and \( n=2 \).

4. Define a rotation of an integer to be an integer with the same digits, in the same order, but possibly rotated in a circular fashion. For example, 3562, 5623, 6235, and 2356 are all rotations of each other. A cyclic number with \( n \) digits has the property that when it is multiplied by 1, 2, 3, 4, ..., \( n \), all the products are rotations of the original number. Find the smallest cyclic number with more than one digit.

5. For \( n \) from 1 to 5, fill in the following table with all the ways to make \( n \) cents from one- and two-cent stamps (where order of the stamps matters). Organize by the number of stamps used. Notice any patterns and explain them.

<table>
<thead>
<tr>
<th>Cost</th>
<th>1c</th>
<th>2c</th>
<th>3c</th>
<th>4c</th>
<th>5c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 stamp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 stamps</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 stamps</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 stamps</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 stamps</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL#</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. Think of a reasonable definition for the sum of an infinite number of terms. Try it on:

   a. \(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\)
   b. \(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots\)
   c. \(\frac{0}{1} + \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots\)

7. Draw, and count the number of edges in, an \(n\)-dimensional hypercube for \(n = 0\) (point), \(n = 1\) (line), \(n = 2\) (square), \(n = 3\) (cube), and \(n = 4\). Can you find a general formula?

8. Write Pascal’s triangle by starting with a 1 in the top spot. For each cell in following rows, add the number above and to the left to the number above and to the right, substituting a zero if either number is not present. For example,

   \[
   \begin{array}{cccccc}
   & & & 1 & & \\
   & & 1 & 1 & & \\
   & 1 & 2 & 1 & & \\
   1 & 3 & 3 & 1 & & \\
   1 & 4 & 6 & 4 & 1 & \\
   \end{array}
   \]

   Let \(c_0=1\), \(c_1=2\), \(c_2=6\), ... be the terms on the central column of Pascal’s triangle.

   a) Each \(c_i\) represents some binomial coefficient \(\binom{n}{k}\), the number of ways to choose a subset of \(k\) items from a collection of \(n\) items, without replacement. Find \(n\) and \(k\) in terms of \(i\).

   b) Evaluate the first few values of the sequence

   \[
   c_0, c_0 c_1, c_0 c_1 + c_1 c_0, c_0 c_1 + c_1 c_0, c_0 c_2 + c_1 c_1 + c_2 c_0, c_0 c_3 + c_1 c_2 + c_2 c_1 + c_3 c_0, \cdots
   \]

   Notice any patterns and explain them.

**Selected References**