Introduction to Counting V

January 11, 2012

Joshua Zucker, joshua.zucker@stanfordalumni.org

Dominoes

1. How many ways are there to tile a $2 \times n$ rectangle with 2×1 dominoes?

2. How many ways are there to tile a $3 \times n$ rectangle with dominoes?

3. How many ways are there to tile a $4 \times n$ rectangle with dominoes?

4. Is there a humanly-reasonable way to extend this to larger rectangles? Could we figure out the number of tilings of an 8×8 chessboard in a reasonable amount of time?

5. What rectangles can have "fault-free" tilings—that is, tilings where the whole rectangle is tiled without having any way to split it into two separate tiled rectangles?

Trominoes

6. How many ways are there to tile a $1 \times n$ rectangle with 3×1 trominoes?

7. How many ways are there to tile a $2 \times n$ rectangle with 3×1 trominoes?

8. How many ways are there to tile a $3 \times n$ rectangle with 3×1 trominoes?

9. How many ways are there to tile a $4 \times n$ rectangle with 3×1 trominoes?

10. Repeat the previous problems using an "L" tromino instead of the straight tromino.

Tours - one step at a time

"One step at a time" means we can travel one space horizontally or vertically on each move.

11. How many ways are there to begin on one square of a $2 \times n$ board, visit every square exatly once, and return to your starting point? These are called *closed* tours.

12. How many ways are there to begin on one corner of a $2 \times n$ board and visit each square exactly once? These are called *open* tours.

13. Are these questions manageable for $3 \times n$ boards?