

Russian-style Session Problems

1. Find all pairs of integers (x, y) such that $xy = x + y$.
2. What is the last (unit) digit of the number 7^{7^7} ?
3. If the point of intersection of the altitudes of a triangle is reflected across each side of the triangle the resulting three points all lie on the circumscribed circle. Prove it.
4. Find a four-digit number which is a perfect square and such that its first two digits are the same, and its last two digits are the same, too.
5. Find the sum $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + k \cdot k!$.
6. Can the altitudes of a triangle have lengths 1, 2, and 3?